1060, Fa 13
Day 4

Functions: the language of Calculus

$f(x) = x^2 + 2x + 1$

means

$f(\boxed{\text{thing}}) = (\boxed{\text{thing}})^2 + 2\cdot(\boxed{\text{thing}}) + 1$

Can simplify or evaluate using algebra

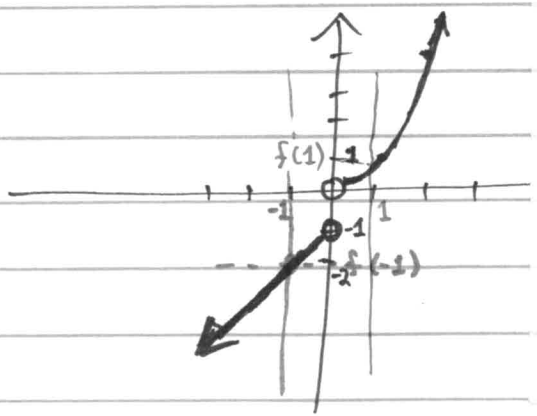
the Domain of f is the set of [input #'s x] where f(x) is defined

the Range of f is the set of [output #'s y]
i.e. the #'s y s.t. $f(x) = y$ for some input x

to graph a function

plot the point $(x, f(x))$
for each x in the domain of f
(↑ where f is defined)

Eg: Graph $f(x) = \begin{cases} x-1 & \text{if } x<0 \\ x^2 & \text{if } x>0 \end{cases}$



Carefully Explain this!

to graph $x-1$ if $x<0$
step 1: graph $x-1$
step 2: erase the parts above $x=0$

to graph x^2 if $x>0$
step 1 } similar
step 2 }

10 we'll need to be able to evaluate functions by plugging in #'s.
we'll also need to know ...

How to Read a Graph:

to find $f(x)$
① go to x
② draw a vertical line through x
③ go over from the point of intersection to find $f(x)$

Eg: find $f(1)$ = follow 3 steps = 1

$f(-1)$ = follow 3 steps = -2

[redacted]

each input has AT MOST ONE output

Notice: the way we read graphs gives us

The Vertical Line Test

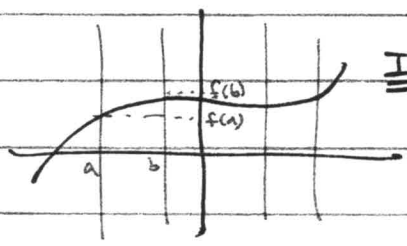
If f is a function

$\Leftrightarrow$

every vertical line intersects its graph at most once

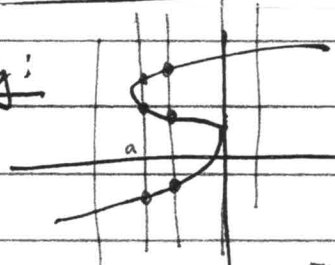
2-5

Eg:



Is a function

Eg:



Is Not a function

[redacted] $f(a)$ Is ILL-DEFINED

"3 ft of vine costs $1, $5, or $15"
is very bad

In our Example Above:

$f(x)$ is defined $\Leftrightarrow$ $x \neq 0$ (graph of f is above x [redacted] $\Leftrightarrow x \neq 0$)

$\Rightarrow$ Domain of $f = (-\infty, 0) \cup (0, \infty)$

5

$f(x)$ hits all $y > 0$
and all $y < -1$

$\Rightarrow$ Range of $f = (-\infty, -1) \cup (0, \infty)$

Eg:

$$f(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$$

① Graph Carefully

② examine Domain

③ examine Range

Ranges are tricky... you often need to think about it
can often find domains algebraically.

Eg:

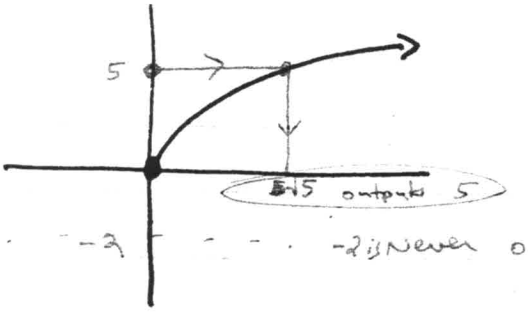
Let $f(x) = \sqrt{2x}$

$\sqrt{2x}$ is defined $\Leftrightarrow 2x \geq 0 \Leftrightarrow x \geq 0$

The domain of f is $[0, \infty)$

- - - - - - - - - - - - - -

to find the ranges sketch!

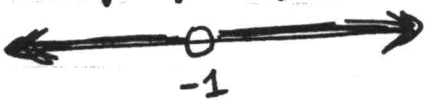


this outputs all y's $y \geq 0$
$\Rightarrow$ Range is $[0, \infty)$

Eg: $f(x) = \frac{1}{x+1}$

$\frac{1}{x+1}$ is defined $\Leftrightarrow x+1 \neq 0$
$\Leftrightarrow x \neq -1$

$\Rightarrow$ The domain of f is



Which is $(-\infty, -1) \cup (-1, \infty)$.